Noncompact self-shrinkers for mean curvature flow with arbitrary genus

Reto Buzano

In his lecture notes on mean curvature flow, Ilmanen conjectured the existence of noncompact self-shrinkers with arbitrary genus. Here, we employ min-max techniques to give a rigorous existence proof for these surfaces. Conjecturally, the self-shrinkers that we obtain have precisely one (asymptotically conical) end. We confirm this for large genus via a precise analysis of the limiting object of sequences of such self-shrinkers for which the genus tends to infinity. Finally, we present some numerical evidence for a further new family of noncompact self-shrinkers with odd genus and two asymptotically conical ends.

This is joint work with Huy Nguyen and Mario Schulz.

Università di Torino (Italy).

Cheeger-type inequality for differential forms

Gilles Courtois

On a compact Riemannian manifold, the Cheeger's inequality relates the first non zero eigenvalue of the Laplacian of functions with an isoperimetric constant of the manifold. J. Cheeger asked if an analogous inequality would hold for the first non zero eigenvalue of differential forms. We will discuss the case of 1-differential forms.

Joint work with Adrien Boulanger.

Sorbonne Université - CNRS (France).

Equivariant min-max theory to construct free boundary minimal surfaces in the unit ball

Giada Franz

A free boundary minimal surface (FBMS) in the three-dimensional Euclidean unit ball is a critical point of the area functional with respect to variations that constrain its boundary to the boundary of the ball (i.e., the unit sphere). A very natural question is whether there are FBMS in the unit ball of any given topological type.

In this talk, we will present the construction of a family of FBMS with connected boundary and arbitrary genus, via an equivariant version of Almgren-Pitts min-max theory à la Simon-Smith. We will see how this method enables to control the topology of the resulting surfaces and also to obtain information on their index.

ETH Zürich (Switzerland).

A nonlinear spectrum on closed manifolds

Christos Mantoulidis

The p-widths of a closed Riemannian manifold are a nonlinear analogue of the spectrum of its Laplace–Beltrami operator, which was defined by Gromov in the 1980s and corresponds to areas of a certain min-max sequence of hypersurfaces. By a recent theorem of Liokumovich–Marques–Neves, the p-widths obey a Weyl law, just like the eigenvalues do. However, even though eigenvalues are explicitly computable for many manifolds, there had previously not been any ≥ 2-dimensional manifold for which all the p-widths are known. In recent joint work with Otis Chodosh, we found all p-widths on the round 2-sphere and thus the previously unknown Liokumovich–Marques–Neves Weyl law constant in dimension 2. Our work combines Lusternik–Schnirelmann theory, integrable PDE, and phase transition techniques.

Rice University (USA).

A quasiconformal Hopf soap bubble theorem

Pablo Mira

We show that any compact surface of genus zero in Euclidean 3-space that satisfies a quasiconformal inequality between its principal curvatures is a round sphere. This solves an old open problem by H. Hopf, and gives a spherical version of Simon's quasiconformal Bernstein theorem.

Joint work with J.A. Galvez and M.P. Tassi.

Universidad Politécnica de Cartagena (Spain).

Minimizing combinations of Laplace eigenvalues and applications

Romain Pétrides

We give a variational method for existence and regularity of metrics which minimize combinations of eigenvalues of the Laplacian among metrics of unit area on a surface.

We show that there are minimal immersions into ellipsoids parametrized by eigenvalues, such that the coordinate functions are eigenfunctions with respect to the minimal metrics.

As one of the applications, we explain a new method to construct non-planar minimal spheres into 3d-ellipsoids after Haslhofer-Ketover and Bettiol-Piccione.

Université de Paris (France).

Nondegenerate minimal submanifolds as energy concentration sets

Alessandro Pigati

In the last two decades, various energies of physical significance have been shown to effectively approximate the area functional. These energies, which are defined (roughly speaking) on the set of functions on a given ambient manifold, tend to concentrate near the zero set of the function. By taking natural spatial rescalings, it turns out that the zero level sets of a sequence of maps converge to a submanifold of bounded area (a Gamma-convergence result), and that for critical maps the limit is minimal, i.e., critical for the area (the relevant notion of limit is different in the two settings, though). This is the case for the Allen-Cahn energy (in codimension one), the Ginzburg-Landau energy (in codimension two) and the Yang-Mills-Higgs energy for $U(1)$-bundles (again in codimension two).

Here we look at the converse problem: given an ambient (closed, oriented) Riemannian manifold and any nondegenerate minimal submanifold M, under the appropriate topological assumptions we show that M always arises as such a limit for a sequence of critical maps. The strategy we employ is entirely variational; it relies on an axiomatic framework relating Gamma-convergence to min-max problems, developed by Jerrard and Sternberg, and generalizes a recent work for geodesics (by Colinet, Jerrard, and Sternberg), which uses the same strategy. Among other ingredients, we characterize nondegenerate minimal submanifolds as strict local minimizers for a deformation of the mass, with respect to the flat norm on cycles, and we prove a new rigidity result for nondegenerate minimal submanifolds among rectifiable varifolds.

(Joint work with Guido De Philippis)

New York University (USA).

The Allen-Cahn equation on surfaces: looking for solutions

Frank Pacard

École Polytechnique (France).

Area variations under Lagrangian constraints

Tristan Rivière

Abstract : The parametric approach to the resolution of the Willmore conjecture introduced by the author some months ago has increased even more the motivation for studying the variations of the area of surfaces under the lagrangian constraint in the Grassman Manifolds $G_2(\mathbb{R}^4)$ of oriented 2-planes in $\mathbb{R}^4$ and their legendrian lifts in the Stiefel S^1-bundle $V_2(\mathbb{R}^4)$ of orthonormal 2-frames in $\mathbb{R}^4$. The goal of this talk is to present some of the difficulties posed by these pointwize constraints and some of the answers that can be given. We will in particular derive a new almost monotonicity formula satisfied by critical points of the area under legendrian constraint in Sasaki-Einstein 5-manifolds (which can be seen as a conservation law for an "extreme anisotropic" variational problem). In the second part of the talk we will propose an approach to the Willmore minimization in Lagrangian Integral Homology Classes of Calabi Yau and Kähler-Einstein Surfaces in relation with some new S^1-harmonic map variational theory. The link between the two parts of the talk will be made.

ETH Zürich (Switzerland).

Homotopic Plateau-Douglas problem

Elefterios Soultanis

The Plateau-Douglas problem generalizes Plateau's famous problem and asks to find an area minimizing (weakly conformal) map spanning k given curves (inside a given ambient space) from a surface with k boundary components and given genus. In this talk I will describe the homotopic variant of this problem, where the area minimizer is subject to further topological restrictions. I will describe the relevant topological data, namely 1-homotopy classes, and discuss the minimization problem in a metric space setting where no smooth structure is available.

Radboud University (Netherlands).

On the area of Lawson minimal surfaces in the 3-sphere

Martin traizet

Lawson has constructed, for each positive genus g, a closed minimal surface in the sphere $\mathbb{S}^3$ whose area is less than 8π. I will explain how to construct these surfaces by integrable system methods (DPW) when the genus g is large. This approach yields fine estimates for their area. Suprisingly, the asymptotic expansion of the area when the genus goes to infinity involves the value of Riemann's zeta function at $z = 3$. I will explain the path from minimal surfaces in $\mathbb{S}^3$ to values of the zeta function.

Joint work with Lynn Heller, Sebastian Heller and Steven Charlton.

Université de Tours (France).